

## COMMENTS ON THE PAPER: A PARAMETRIC STUDY OF THE COMPLEX VARIABLE METHOD FOR ANALYZING THE STRESSES IN AN INFINITE PLATE CONTAINING A RIGID RECTANGULAR INCLUSION†

THE writer wishes to compliment the authors for their thorough analysis of this important problem and for the extensive numerical data presented in their paper. It is also the object of this letter to discuss some additional pertinent references which have not been mentioned by the authors.

Heller and co-workers [1] presented a solution for stresses around rectangular openings with rounded corners in a uniformly loaded plate. Numerical results were presented for a wide spectrum of length to width ratios and radii of curvature using the complex variable method of Muskhelishvili in conjunction with an approximate conformal mapping technique. Curves were also given showing the maximum values of the boundary stresses as a function of both aspect ratio and corner radius.

With regards to the transformation functions used in the determination of stresses, it is interesting to point out that the problem has been discussed by several authors. Wittrick [2] has determined approximate transformation functions in the form of three-term polynomials which transform the region outside a square or an equilateral triangle onto the region outside the unit circle. The three coefficients of the polynomial are evaluated using three conditions: size of the square, zero curvature at the center of the side of the square and a given corner curvature. References [3] and [4] can be considered as extensions of Wittrick's method for other boundary shapes and for internal regions respectively.

Wilson [5] developed very general techniques for the determination of mapping functions and calculation of stresses in the case of noncircular holes. The main goal of Wilson's study was the determination of stresses in solid propellant rocket motor grains (as a first approximation they are considered as long, circular cylinders with a star perforation).

Reference [6] presents an interesting application of the Schwarz-Christoffel transformation to star-shaped domains.

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† By C. S. CHANG and H. D. CONWAY, *Int. J. Solids Struct.* **4**, 1057-1066 (1968).

### REFERENCES

- [1] S. R. HELLER, J. S. BROCK and R. BART. The stresses around a rectangular opening with rounded corners in a uniformly loaded plate. *Proc. 3rd U.S. natn. Congr. appl. Mech.* 357-368 (1958).
- [2] W. H. WITTRICK, Some simple transformation functions for square and triangular holes with rounded corners. *Aeronaut. Q.* **11**, 195-199 (1960).

- [3] P. A. LAURA, Analytical determination of stresses around mine shafts of arbitrary cross section. *Proc. 1st Int. Congr. Rock Mech.*, Lisbon (1966).
- [4] P. A. LAURA and A. J. FAULSTICH, JR., An application of conformal mapping to the determination of the natural frequency of membranes of regular polygonal shape. *Developments in Mechanics*, Vol. 3, Part 2, pp. 155-163; Wiley (1967).
- [5] H. B. WILSON, A method of conformal mapping and the determination of stresses in solid propellant rocket grains, Report No. S-38; Rohm and Haas Company, Huntsville, Alabama (April 1963).
- [6] K. RIM and R. O. STAFFORD, Derivation of mapping functions for star shaped regions, NASA Contractor Report No. NASA CR-192 (March 1965).

### AUTHORS' CLOSURE

THE authors wish to thank Professor Laura for his kind comments and for bringing their attention to the additional references.

In the continuation study on composite materials the authors are using Kikukawa's method for evaluating the mapping functions [1, 2]. It has been found that this transformation is more flexible with respect to geometry and also very suitable for machine computation. The results of this study will be presented in forthcoming papers.

### REFERENCES

- [1] J. H. GOODIER and P. G. HODGE, JR., *Elasticity and Plasticity*, pp. 45-47. Wiley (1958).
- [2] M. KIKUKAWA, On plane-stress problems in domains of arbitrary profiles. *Proc. 4th Japan natn. Congr. appl. Mech.* pp. 149-152 (1954).